## 7.4, 7.9 - Currents in the BJT, HBTs

HBT fabrication more complicated than slapping together $p+n p$ materials...

What is different than CMOS here? Think $\mathbf{R}=\rho \mathrm{L} / \mathrm{A}(\Omega)$


- Review this slide, and everything must make sense, else go back and review part A of the previous lecture!

(1) Holes injected do what? diffuse across EB
(2) Holes reach BC and do what? drift to $C$

Emitter (inject holes)
(3) Holes injected do what? recombine with $B$ electrons

Base (historical, Ge slab)
(4) Electrons injected do what? recombine with $B$ holes

Collector (collect holes)
$\mathrm{I}=\mathrm{I}_{\mathrm{E}}=\mathrm{I}_{\mathrm{B}}+\mathrm{l}_{\mathrm{C}}$
(5) Electrons injected do what? diffuse across EB
(6) Reverse bias e or h do what? drift across BC (small)

Remember: $\quad p+n$ for $E B$ so (1) $\gg$ (5), $W_{b} \ll L_{p}$ so (2) >> (3), but (3) $\neq 0$

- Goal today, calculate currents for the BJT
- Key Assumptions...

Holes diffuse from emitter to collector, drift is negligible (no E-field in B).
$\mathrm{Y}=1 \ldots$ ( $\mathrm{i}_{\mathrm{E}}$ is all holes).
No collector reverse saturation current (6).
$E B$ and $B C$ junctions have the same area in one dimension (i.e. all horiz. current in diagram...).

All current and voltages steady state.

Things will get VERY complicated, but hang on, I promise I will make them VERY simple in the end!

- Recall minority currents in a $p+n$ junction (part of our p+np BJT).

$$
\delta p\left(x_{n}\right)=\Delta \underbrace{p_{n} e^{-x_{n} / L_{p}}}_{\Delta p_{n}=p_{n}\left(e^{q V / k T}-1\right)}
$$

- Review,

The diagram at right is if V is positive...
If $\mathrm{V}=0$ it will look like what?

If V is negative, will look like what?
The equation above predicts the answer! See the drawing I do at right during the video.

- Forward bias, diff dominates...

note above is a logarithmic scale...
- Note Fermi Levels shifts as voltage is applied...

- Based on what we learned for the previous slide, where in the base, for the diagram at right should we see an excess of holes, or no (zero) holes at all?
- Apply the previous approach to both EB and $B C$ junctions to get excess hole concentration at base edges:

$$
\begin{array}{ll}
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right) & \text { labeled } \\
\Delta p_{C}=p_{n}\left(e^{q V_{C B} / k T}-1\right) & \text { at right }
\end{array}
$$

- Under normal conditions we can simplify these equations to the following, why? Answer based on equations and the diagram......

$$
\begin{aligned}
& \Delta p_{E} \approx p_{n} e^{q V_{E B} / k T} \\
& \Delta p_{C} \approx-p_{n}
\end{aligned}
$$




- Apply diffusion equation in the base... why?

$$
\frac{d^{2} \delta p\left(x_{n}\right)}{d x_{n}^{2}}=\frac{\delta p\left(x_{n}\right)}{L_{p}^{2}}
$$

general meaning of diffusion equation: smaller $L p$, or larger $\Delta p$, results in more curvature (rate of change in slope, of the change in charge)...


- How to solve a differential equation:
(1) Here is the equation
(2) Put
derivatives in rank order
(3) Look up online or in a math book what type of diff. equation it is.

$$
\frac{d^{2} \delta p\left(x_{n}\right)}{d x_{n}^{2}}=\frac{\delta p\left(x_{n}\right)}{L_{p}^{2}}
$$

$$
\frac{d^{2} \delta p\left(x_{n}\right)}{d x_{n}^{2}}-\frac{\delta p\left(x_{n}\right)}{L_{p}^{2}}+0=0
$$

$$
y^{\prime \prime}+a y^{\prime}+b y=0
$$

$$
\left[a^{2}-4 b\right]>0 ? \Rightarrow\left[0^{2}-4\left(-1 / L_{p}^{2}\right)\right]>0, \text { yes! }
$$

The book (Kreyszig, 2.2) tells you the general solution! Is a $2^{\text {nd }}$ order homogeneous differential equation with constant coefficients and two real roots :

$$
\delta p\left(x_{n}\right)=C_{1} e^{x_{n} / L_{p}}+C_{2} e^{-x_{n} / L_{p}}
$$

(4) Last step, apply boundary conditions (a.k.a. common
 sense) to solve for unknowns in the general solution.

In Chapter 4 we assumed one of the constants $\left(C_{1}\right)$ was zero since excess holes disappeared at long distance ( $x$ ) into n-type slab. Common sense! Hole concentration can't go to infinity!
But... we cannot do that here since $W_{b} \ll L_{p} \ldots$ Hmm... next slide!

$$
\Delta p_{E}-\begin{array}{ll}
\text { this plot just a guess, } \\
\text { we have to prove... }
\end{array} \begin{aligned}
& \Delta p_{E} \approx p_{n} e^{q V_{E B} / k T} \\
& \Delta p_{C} \approx-p_{n}
\end{aligned}
$$

- Lets apply boundary conditions (this is easy!). We know concentrations at edges (see above) and we have this equation now:

$$
\begin{aligned}
& \delta p\left(x_{n}\right)=C_{1} e^{x_{n} / L_{p}}+C_{2} e^{-x_{n} / L_{p}} \\
& \delta p\left(x_{n}=0\right)=C_{1}+C_{2}=\Delta p_{E} \\
& \delta p\left(x_{n}=W_{b}\right)=C_{1} e^{W_{b} / L_{p}}+C_{2} e^{-W_{b} / L_{p}}=\Delta p_{C}
\end{aligned}
$$



- For most differential equations, you would be done now... but for BJTs is more complex...
- 2 Eq. and 2 Variables, ICBST solving for $\mathrm{C}_{1}, \mathrm{C}_{2}$ we get:

$$
C_{1}=\frac{\Delta p_{C}-\Delta p_{E} e^{-W_{b} / L_{p}}}{e^{W_{b} / L_{p}}-e^{-W_{b} / L_{p}}}
$$

$$
C_{2}=\frac{\Delta p_{E} e^{W_{b} / L_{p}}-\Delta p_{C}}{e^{W_{b} / L_{p}}-e^{-W_{b} / L_{p}}}
$$

Substitute newly found $\mathrm{C}_{1}, \mathrm{C}_{2}$ back in to: $\quad \delta p\left(x_{n}\right)=C_{1} e^{x_{n} / L_{p}}+C_{2} e^{-x_{n} / L_{p}}$

- The two parts of the equation add up to a ~linear (nearly) hole distribution in base region:



$$
\begin{aligned}
& \delta p\left(x_{n}\right)=M_{1} \Delta p_{E} e^{-x_{n} / L_{p}}-M_{2} \Delta p_{E} e^{x_{n} / L_{p}} \\
& M_{1}=\frac{e^{W_{b} / L_{p}}}{e^{W_{b} / L_{p}}-e^{-W_{b} / L_{p}}} \quad M_{2}=\frac{e^{-W_{b} / L_{p}}}{e^{W_{b} / L_{p}}-e^{-W_{b} / L_{p}}}
\end{aligned}
$$




diffusion (PN forward) diffusion (base) drift (PN reverse)

$$
\delta p\left(x_{n}\right)=M_{1} \Delta p_{E} e^{-x_{n} / L_{p}}-M_{2} \Delta p_{E} e^{x_{n} / L_{p}}
$$

- What causes each change in concentration of carriers? See below diagram...
- Why don't we show excess holes in C?
... because these are minority carriers for each region.
- Looks like two diodes! One forward bias, one reverse, and then with profiles joined at the base!
- We know at $x_{n}=0$ we have all $I_{\text {Ep }}$ (we assumed $\mathrm{Y}=1$ )
- We know at $x_{n}=W_{b}$ we have all $I_{C}$
- We now have an equation $\left(\delta_{p}\right)$ for hole density at each of these edges
- We can then solve for $I_{\text {Ep }}$ and $I_{C}$ using Eq. 422 from Ch. 4,

$$
I_{p}\left(x_{n}\right)=-q A D_{p} \frac{d \delta p\left(x_{n}\right)}{d x_{n}}
$$


-What is this equation and what does it tell us?

$$
\begin{gathered}
\delta p\left(x_{n}\right)=C_{1} e^{x_{n} / L_{p}}+C_{2} e^{-x_{n} / L_{p}} \\
\downarrow \\
I_{p}\left(x_{n}\right)=-q A D_{p} \frac{d \delta p\left(x_{n}\right)}{d x_{n}}
\end{gathered}
$$

- We assumed $\mathrm{y}=1$, so at $\boldsymbol{x}_{n}=0$ our $\mathrm{I}_{\mathrm{Ep}}$ current is equal to:

$$
I_{E p}=I_{p}\left(x_{n}=0\right)=q A \frac{D_{p}}{L_{p}}\left(C_{2}-C_{1}\right)
$$

- We know at $x_{n}=W_{b}$ our $I_{C}$ current is equal to:
$I_{C}=I_{p}\left(x_{n}=W_{b}\right)=q A \frac{D_{p}}{L_{p}}\left(C_{2} e^{-W_{b} / L_{p}}-C_{1} e^{W_{b} / L_{p}}\right)$
- So like before, we can solve for $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ with 2 eq. and 2 variables (but lets skip the math, you'll see why).

- Solving for $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ (skip this fun math) we can obtain:

$$
\begin{aligned}
& \boldsymbol{\lambda} I_{E_{p}}=q A \frac{D_{p}}{L_{p}}\left(\Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}}-\Delta p_{C} \operatorname{csch} \frac{W_{b}}{L_{p}}\right) \\
& \Delta I_{C}=q A \frac{D_{p}}{L_{p}}\left(\Delta p_{E} \operatorname{csch} \frac{W_{b}}{L_{p}}-\Delta p_{C} \operatorname{ctnh} \frac{W_{b}}{L_{p}}\right)
\end{aligned}
$$



- But how will I get $\mathrm{I}_{\mathrm{B}}$ ? Is easy... think networks.

$$
I_{B}=I_{E p}-I_{C}
$$

$$
=q A \frac{D_{p}}{L_{p}}\left[\left(\Delta p_{E}+\Delta p_{C}\right)\left(\operatorname{ctnh} \frac{W_{b}}{L_{p}}-\operatorname{csch} \frac{W_{b}}{L_{p}}\right)\right]
$$

$$
=q A \frac{D_{p}}{L_{p}}\left[\left(\Delta p_{E}+\Delta p_{C}\right) \tanh \frac{W_{b}}{2 L_{p}}\right]
$$

That's it, here are the equations to predict currents for each terminal of a BJT! Hang on... in next lecture I will make them elegantly simple for you!

- How do you solve a differential equation? (Just tell me the four general steps).
- The hole concentration across the base, what does it look like?
- Is zero everywhere.
- Is excess on the emitter-side and zero at the collector-side.
- Is zero on the emitter-side and excess at the collector-side.
- I am tired and am going to bed...
- Current is driven from emitter into the base by: drift, diffusion, neither, both.

$$
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right)
$$ both.

- Current is driven from base into the collector by: drift, diffusion, neither, both.

$$
I_{E p} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}}
$$

- Lastly, peak ahead, notice how in the equations we derived, they all share a blue-highlighted term such that all current increase or decrease at the SAME rate (but are obviously not equal). You should have expected this already based on what we know about BJTs...
- With a bit more work, I can get to the following simplified equations:

$$
\frac{\sqrt{E p} I_{E} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}}}{\sqrt{I_{C}} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{csch} \frac{W_{b}}{L_{p}}} \frac{I_{B} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \tanh \frac{W_{b}}{2 L_{p}}}{\sqrt{2}}
$$

- Remember, this is the equation for $\Delta \mathrm{p}_{\mathrm{E}}\left(\mathrm{V}_{\mathrm{EB}}\right)$ but what does this mean?


$$
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right) \quad \mathrm{V}_{\mathrm{EB}} \uparrow \quad \mathrm{I}_{\mathrm{ED}} \& \mathrm{I}_{\mathrm{C}} \& \mathrm{I}_{\mathrm{B}} \uparrow
$$

But what is the other 'stuff'...

They ALL INCREASE the same with $V_{E B}$, diode forward bias, which makes perfect sense!

$$
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right)
$$

- How about this term in front? Steal the $p_{n}$ from $\Delta p_{E} \ldots$ recognize it?

$$
q A \frac{D_{p}}{L_{p}} p_{n}
$$

$$
\rangle I_{E_{p}} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}}
$$

- Why does it only deal with the base $\left(\mathrm{p}_{\mathrm{n}}\right)$ ? is $p+n$ diode, and also that is the whole point of why we derived these by solving the diff. eq. in the base only!
... this is getting simpler!
... last thing we need is a way to differentiate between the three components...


$$
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right)
$$

$\downarrow I_{E p} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}} \longrightarrow \mathbf{c t n h}=\frac{1}{\tanh }=\frac{e^{2 x}+1}{e^{2 x}-1} \longrightarrow \sim 100$
$\downarrow I_{C} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{csch} \frac{W_{b}}{L_{p}} \longrightarrow \mathbf{c s c h}=\frac{1}{\sinh }=\frac{2}{e^{x}-e^{-x}} \longrightarrow \sim 100$
$\downarrow I_{B} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \tanh \frac{W_{b}}{2 L_{p}} \longrightarrow \tanh =\frac{\sinh }{\cosh }=\frac{e^{2 x}-1}{e^{2 x}+1} \longrightarrow \sim 0.005$

- Lastly, what do these hyperbolic trig functions do?
- Remember, we want Wb smaller than Lp for good design (so holes get across without recombining)!

$$
\frac{W_{b}}{L_{p}} \approx 0.1 \quad \text { to } 0.001 \quad e^{0.01}=1.01
$$

Example 7-4 in book:
$N_{D}=10^{15} / c c$
$L_{P}=108 \mu \mathrm{~m}$
$W_{B}=1 \mu \mathrm{~m}$
$\beta=832$

$$
\begin{gathered}
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right) \\
I_{E p} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}} \\
I_{C} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{csch} \frac{W_{b}}{L_{p}} \\
I_{B} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \tanh \frac{W_{b}}{2 L_{p}}
\end{gathered}
$$

, Review 'parts' one last time...
Reverse sat. current (constant)!
Effect of $\mathrm{V}_{\mathrm{EB}}$ ! ALL scale together!
Current magnitudes must be different, and the effect of $W_{b}$ and $L_{p}$ !


20 - Simpler Formula for $I_{B}$
UNIVERSITY OF
$\Rightarrow I_{B} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \tanh \frac{W_{b}}{2 L_{p}}$
for small $W_{b} / L_{p}$ (good design) via series expansion it can be shown that...

$$
I_{B} \approx \frac{q A W_{b} \Delta p_{E}}{2 \tau_{p}}
$$

- What does this equation say? Look at geometry for Qp...

$$
\Rightarrow \quad Q_{p}=\frac{1}{2} q A \Delta p_{E} W_{b} \quad I_{B} \approx \frac{Q_{p}}{\tau_{p}}
$$




- To simplify our calculations we assumed $\delta=1$ (which is a safe assumption).
- However, if you are a designer, you know in practice it is not unity, and would like to know how to get it as close to unity as possible...

$$
\begin{gathered}
\gamma=\frac{i_{E p}}{i_{E n}+i_{E p}} \\
\gamma=\left[1+\frac{L_{p}^{n} n_{n} \mu_{n}^{p}}{L_{n}^{p} p_{p} \mu_{p}^{n}} \tanh \frac{W_{b}}{L_{p}^{n}}\right]^{-1} \approx\left[1+\frac{W_{b} n_{n} \mu_{n}^{p}}{L_{n}^{p} p_{p} \mu_{p}^{n}}\right]^{-1}
\end{gathered}
$$


$L_{p}^{n}=$ hole diffusion length in n-type base $\quad \operatorname{sech}(z)=1 / \cosh (z)$
$\mu_{n}^{p}=$ electron mobility in p-type emitter

- Also base transport should be as close to unity as possible for good design

$$
i_{c}=B i_{E p}
$$

$$
B=\frac{I_{C}}{I_{E}}=\frac{q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{csch} W_{b} / L_{p}}{q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} W_{b} / L_{p}}=\operatorname{sech} \frac{W_{b}}{L_{p}}
$$

- It should be noted that the highest performance BJTs are Heterojunction BTs (HBTs) using a wider-band-gap material for the emitter, why?


$$
\gamma=\frac{i_{E p}}{i_{E n}+i_{E p}}
$$


$\Delta$ Eg effects exponential Fermi distribution of electrons that can diffuse over the barrier!

$$
\frac{I_{p}}{I_{n}} \propto \frac{N_{A}^{E}}{N_{D}^{B}} e^{\Delta E_{g} / k T}
$$

- HBTs use materials such as AIGaAs/GaAs (which is AIGaAs)?
- To get the current equations, we solved for what in the base? Drift current equation, or diffusion current equation, neither, or both?
- For our equations, what is the yellow? How does it effect currents as voltage changes?
- Is it good that all currents respond linearly to base current? Not that I can think of, or wow I now have a single device that linearly amplifies current!
- For our equations, what is the blue? Hint, the BJT is just diodes, and this is a key part of the diode equation...
- For our equations, what is the pink? How does it effect currents? Hint, I need these, without these would I still have an amplifier?
- If we made $\mathrm{W}_{\mathrm{b}}$ really large, what would our circuit and equations reduce to? You can do the math, but to make this easier just trust your instincts, if Wb becomes large, you basically get two separate diodes, not a BJT. Does that give you amplification then?
- Why do we make HBTs? Improves emitter injection efficiency, but how?


$$
\Delta p_{E}=p_{n}\left(e^{q V_{E B} / k T}-1\right)
$$

$$
\begin{aligned}
& \sqrt{E p} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{ctnh} \frac{W_{b}}{L_{p}} \\
& I_{C} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \operatorname{csch} \frac{W_{b}}{L_{p}} \\
& I_{B} \approx q A \frac{D_{p}}{L_{p}} \Delta p_{E} \tanh \frac{W_{b}}{2 L_{p}}
\end{aligned}
$$

- If time allows, let's run through an example HBT fabrication process...

An InP-Based Optoelectronic Integrated Circuit for Optical Communication Systems
Master of Science in Electrical Engineering Research Thesis
Shraga Kraus
Submitted to the Senate of the Technion - Israel Institute of Technology lyar 5766 Haifa May 2006


Figure 2.10: Wafer layers after emitter etch


Figure 2.11: Wafer layers after base metal mask


Figure 2.13: Wafer layers after collector etch



Figure 2.12: Wafer layers after base etch


Figure 2.14: Wafer layers after collector metal implementation


Figure 2.15: Wafer layers after subcollector etch


Figure 2.16: Wafer layers after transistor passivation. Only emitter via is shown in this cross-section


Figure 2.17: Metal 1 as connected to transistor contacts. Only emitter connection is shown in this cross-section

(a)

(c)

(e)

(b)

(d)

(f)

Figure 2.19: SEM images of a transistor at various fabrication process steps: (a) emitter etch (b) base metal deposition and liftoff (c) emitter protect (d) collector protect (e) isolation (f) emitter expose

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(a)

(c)

(b)

(d)

Figure 2.20: SEM images of a transistor and circuit at various fabrication process steps: (a) base-collector via (b) metal 1 deposition and liftoff (c) completed capacitor (d) completed interconnects with crossovers and a resistor

28 ■ Testing!


Figure 2.46: Ebers-Moll model (injection version)

- Amplification factor is $15 \mathrm{~mA} / 200 \mu \mathrm{~A}=75$
- We will talk about the circuit model above next time... looks simple right?

(a)

(b)

Figure 2.21: DC measurement setup: (a) measurement system (b) handmade probes


Figure| 2.23: Common emitter measurement curves (measured on a large area device). $I_{B}$ varies from 0 to $200 \mu \mathrm{~A}$ in $20 \mu \mathrm{~A}$ steps

